Comparing expressive power in two-dimensional semantics

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0. The bigger picture

Variables explained away — Quine (1960)

(0a) $(\exists x)\ [P(x) \wedge Q(x)]$ vs. $P \cap Q \neq \emptyset$

(b) $(\exists w)\ [w_0 R w \wedge p(w)]$ vs. $\Diamond\ p$

Explaining index variables away — Köpping & Zimmermann (forthcoming)

Whether two-dimensional logic is expressively equivalent to intensional logic is open to interpretation (and ideology).

Propositionalism — Quine (1953); D. Kaplan (1975); Larson (2002)

Intensionality is (reducible to) clausal embedding.

Law of the instrument — A. Kaplan (1964: 28)

Give a small boy a hammer, and he will find that everything he encounters needs pounding.

1. Comparative Expressivity of Formal Languages

Schematic definitions

- A language L^* is *at least as expressive as* a language L iff for any (relevant) expressions α in L there is a (relevant) expression α^* in L^* such that $\alpha^* \sim \alpha$.

where '~' denotes model-theoretic equivalence, i.e.:

- $\alpha^* \sim \alpha$ iff $[\![\alpha^*]\!]^{\vec{d}^*} = [\![\alpha]\!]^{\vec{d}}$

… for all L-determinants $\vec{d}$ and matching L^*-determinants $\vec{d}^*$.

Examples

#	*L*	*L**	relevant expressions	determinants	reversible?
1	1st order logic	pred. functor logic	(closed) sentences	structures	+
2	2nd order logic	PFL2	(closed) sentences	structures	+
3	modal prop. logic	1st order logic	formulae	pointed structures	–
4	modal prop. logic	mon. 2nd order logic	formulae	frames	–
5	int. type logic	2-sorted type theory	typed terms	pointed models	–
6	2-sorted type theory	int. type logic	(closed) sentences	structures + $g(i_0)$	+
7	2-sorted type theory	int. type logic	intensional terms	structures + $g(i_0)$	+

$\alpha \in L$	$\alpha^* \in L^*$	cf.
(1) $(\exists x)\ [P(x) \wedge Q(x)]$	$\mathbf{ERK}PQ$	Quine (1960)
(2) $(\exists P)\ (\forall x)\ [P(x) \wedge \neg P(x)]$	$\mathbf{E}_1\mathbf{NE}_0\mathbf{NR}_0\mathbf{R}_1\mathbf{K}\mathit{PRED}\mathbf{N}\mathit{PRED}$	Dosen (1988)
(3) $\Diamond\ [p \wedge q]$	$(\exists w)\ [w_0 R w \wedge [p(w) \wedge q(w)]]$	Fine (1975)
(4) $[p \rightarrow \Diamond\ p]$	$(\forall w)\ wRw$	van Benthem (1984)
(5) $[\lambda P^{s(et)}.\ (\exists x^e)\ [\mathbf{B}(x) \wedge P\{x\}]]$	$[\lambda P.\ (\exists x)\ [\mathbf{B}(i_0)(x) \wedge P(i_0)(x)]]$	Gallin (1975)
(6) $(\forall f^{s,s})\ (\exists j^s)\ \mathbf{B}(f(j)(x))$	$(\forall R)\ [\Phi(R) \rightarrow (\exists p^{s,t})\ \Sigma(p) \wedge \Diamond[p \wedge \mathbf{B}(x)]]$	

… where Σ abbreviates: $[\lambda p^{s,t}.\ \Diamond[\ \lambda Q^{(s,t),t}.\ [p = {}^\wedge [[\lambda q.\ {}^\vee q] = Q]]](\lambda q.\ {}^\vee q)]$ — Gallin (1975)

… and Φ abbreviates: $[\lambda R.\ (\forall p)\ [\Sigma(p) \rightarrow \Sigma(R(p))]]$

(7) $[\lambda p^{et}.\ [\lambda x^e.\ (\forall j^s)\ [i_0\ \mathbf{Epi}_x\ j \rightarrow p(j)]]]$ — $[\lambda p^{st}.\ [\lambda x^e.\ [\lambda q^{st}.\ \Box\ [{}^\vee q \rightarrow {}^\vee p]]\ (\mathbf{Epi}(x))]]$

Zimmermann (1989)

3. Two-dimensional Languages

Kamp (1971), Montague (1970), Kaplan (1979), Lewis (1980)

Determinants of denotation

$[\![\alpha]\!]^{M,c,i,\ldots}$, where

- *M* is an interpretation (of non-logical constants)
- *c* is a context
- *i* is an index
- '…' could be empty or contain more determinants (e.g. a variable assignment) and will be suppressed

Additional structural assumptions

- Diagonal:
  Each context *c* determines its index i_c due to parameterization:
  $c = (c_1,\ldots,c_n,\ldots,c_k)$, and: $i^c = (i_1^c,\ldots,i_n^c)$.
- No monsters: Kaplan (1989)
  if $^\wedge[\![\alpha]\!]^{M,c} = {}^\wedge[\![\alpha']\!]^{M,c}$ and $^\wedge[\![\beta]\!]^{M,c,i,} = {}^\wedge[\![\beta']\!]^{M,c,i}$, then: $[\![\alpha\beta]\!]^{M,c,i} = [\![\alpha'\beta']\!]^{M,c,i}$,
  where $^\wedge[\![\gamma]\!]^{M,c}$ is the *intension* of γ: $^\wedge[\![\gamma]\!]^{M,c}(i) = [\![\gamma]\!]^{M,c,i}$, for any index *i*.
- … or, equivalently:
  All syntactic constructions are (at most) intensional, i.e.: for every context $c \in C$, there is a corresponding operation Γ_c on (possible) intensions such that for any expression α built up by Σ from expressions β and γ, the following equation holds: $^\wedge[\![\alpha]\!]^{M,c} = \Gamma_c(^\wedge[\![\beta]\!]^{M,c}, {}^\wedge[\![\gamma]\!]^{M,c})$.

Relevant determinants

- *characters* assigning denotations $[\![\alpha]\!]^{M,c,i}$ relative to models *M* and (arbitrary) points of reference (*c*,*i*).

Motivation: linguistic meaning, cognitive significance — Montague (1970), Kaplan (1989)

- *epistemic contents* assigning denotations $[\![\alpha]\!]^{M,c} = [\![\alpha]\!]^{M,c,i^c}$ relative to models *M* and contexts *c*.

Motivation: logical validity; cognitive significance — Montague (1970); Lewis (1979)

- *intensions* assigning denotations $^\wedge[\![\alpha]\!]^{M,c}$ relative to models *M* and contexts *c*.

Motivation: indirect denotation, expressed content — Montague (1970); Kaplan (1989)

Notions of Truth

φ is *true at* (or *in*) a context *c* [relative to a model *M*] iff $[\![\varphi]\!]^{M,c} = 1$.

φ is *true of* an index *i* [relative to a context *c* in a model *M*] iff $^\wedge[\![\varphi]\!]^{M,c}(i) = 1$.

[Hence being true in a context is being true of its index]

φ is *true of* an index-component i_m as the *m*-component [relative to …] iff
$^\wedge[\![\varphi]\!]^{M,c}(c_1,\ldots,i_m,\ldots c_n) = 1$.

4. Properties as Objects of Intentional Attitudes

Propositionalism — cf. Forbes (2001), Montague (2007)

Any intentional attitude is [definable in terms of] a propositional attitude.

Examples

To seek a unicorn is to try for it to be the case that one finds a unicorn. — Quine (1953)

To want chocolate is to desire for it to be the case that one has chocolate. — Larson (2002)

Counterexamples
To think of a unicorn is not to think that there is a unicorn. Montague (1969)
To like chocolate is not to like for oneself to have choocolate. Montague (2007)

Anti-propositionalism
Some intentional attitudes are irreducibly attitudes towards properties. cf. Grzankowski (2013)

Perspectivism Lewis (1979)
Some intentional attitudes are irreducibly attitudes towards properties.

Question
What distinguishes anti-propositionalism and perspecitivism?

Some tentative answers:
The difference between …
… having a property and being exposed to a property
… properties as attributes vs. properties as objects
… truth *at* a location and truth *of* an object

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